Fluxes: A Linguistic Description

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How to refer to Flux and what each way really means.

CONTENTS

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I. PHOTON FLUX

A. Differential photon flux

Differential photon flux will be the main descriptor of the model in question. This is the term:

$$\frac{dN}{dE} = M(E) \quad \left(\mathrm{ph\,cm^{-2}s^{-1}MeV^{-1}}\right) \tag{1}$$

1. Models

This model $M(E)$ can be any of the following words/expressions:

Powerlaw:

$$M(E) = k_0 \left(\frac{E}{E_0}\right)^{\gamma}$$

k_0 = Prefactor ($\mathrm{ph\,cm^{-2}s^{-1}MeV^{-1}}$)
γ = Index
E_0 = PivotEnergy (MeV)

Exponentially cut-off power law:

$$M(E) = k_0 \left(\frac{E}{E_0}\right)^{\gamma} \exp\left(\frac{-E}{E_{cut}}\right)$$

k_0 = Prefactor ($\mathrm{ph\,cm^{-2}s^{-1}MeV^{-1}}$)
γ = Index
E_0 = PivotEnergy (MeV)
E_{cut} CutoffEnergy (MeV)

Super exponentially cut-off power law:

$$M(E) = k_0 \left(\frac{E}{E_0}\right)^{\gamma} \exp\left(-\left(\frac{E}{E_{cut}}\right)^{\alpha}\right)$$

k_0 = Prefactor ($\mathrm{ph\,cm^{-2}s^{-1}MeV^{-1}}$)
γ = Index1
α = Index2
E_0 = PivotEnergy (MeV)
E_{cut} CutoffEnergy (MeV)

Broken power law:

$$M(E) = k_0 \begin{cases} \left(\frac{E}{E_b}\right)^{\gamma_1} & E < E_b \\ \left(\frac{E}{E_b}\right)^{\gamma_2} & \text{otherwise} \end{cases}$$

k_0 = Prefactor ($\mathrm{ph\,cm^{-2}s^{-1}MeV^{-1}}$)
γ_1 = Index1
γ_2 = Index2
E_b= BreakEnergy (MeV)

Smoothly broken power law:

$$M(E) = k_0 \left(\frac{E}{E_0}\right)^{\gamma_1} \left[1 + \left(\frac{E}{E_b}\right)^{\frac{\gamma_1-\gamma_2}{\beta}}\right]^{-\beta}$$

k_0 = Prefactor ($\mathrm{ph\,cm^{-2}s^{-1}MeV^{-1}}$)
γ_1 = Index1
E_0 = PivotEnergy (MeV)
γ_2 = Index2
E_b= BreakEnergy (MeV)
β = BreakSmoothness

Log parabola:

$$M(E) = k_0 \left(\frac{E}{E_0}\right)^{\gamma + \eta \log(E/E_0)}$$

k_0 = Prefactor ($\mathrm{ph\,cm^{-2}s^{-1}MeV^{-1}}$)
γ = Index
E_0 = PivotEnergy (MeV)
η = Curvature

In all of these cases, the 'action' happens with unitless terms - note how the energy variable is divided out by the pivot energy (or cutoff energy, etc), so that the unit of the expression is fully enclosed by the Normalization or Prefactor variable only.

In general, I will use one of two forms $\left(\mathrm{ph\,cm^{-2}s^{-1}MeV^{-1}}\right)$ for Ctools work or $\left(\mathrm{ph\,m^{-2}s^{-1}TeV^{-1}}\right)$ for VERITAS. I will use the CTools version for rest of this note, which is in $\left(\mathrm{ph\,cm^{-1}s^{-1}MeV^{-1}}\right)$.

B. Index

Noting the form of equations above in section I A 1, it is clear that many have an exponent on the energy term. In this case, we are representing the differential photon flux, so the index label can be labeled as 'photon index'.

In some cases, authors or software may refer to 'energy index'; this is the effective exponent on energy term when writing the differential energy flux, shown below in section II A, which is 'photon index' + 1. Note the sign - usually only the number is referred to while the sign is ignored. Energy index is the more positive value, so obviously the number is closer to zero.

Per convention, the index specifies the "softness" or "hardness" of a source. A spectrum that is "softening" has a index that is becoming more negative, while "hardening" is becoming more positive. A soft source, which could have an index > 2, has a very steep spectrum so that the majority of events detected are at the lower energy range of the analysis.

C. Integral photon flux

Integral photon flux is described with the energy bin edges. This is generally used while referring to the instrument - e.g. VERITAS sees Crab at 6 photons min^{-1} means the Crab's integral photon flux with the VERITAS effective area implied in the statement.

$$\int_{E_{min}}^{E_{max}} \frac{dN}{dE} dE = \int_{E_{min}}^{E_{max}} M(E) dE \quad \left(\mathrm{ph\,cm^{-2}s^{-1}}\right) \tag{2}$$

Sometimes, this value is just called the *photon flux*. I prefer not to use this less descriptive form, and will always overdescribe, therefore this is the *integral photon flux*. In certain contexts, the unit for this integral photon flux will still contain the MeV^{-1} because it is to make clear that it is for a certain energy bin; this is unclear and should be avoided because it can be confused with the units for differential photon flux.

If using the VERITAS units $\mathrm{ph\,m^{-2}s^{-1}TeV^{-1}}$, simply multiply this evaluated result by $(1 \times 10^{-4}) \times (1 \times 10^{-6})$ to convert cm to meters and MeV to TeV.

II. ENERGY FLUX

The energy flux is a more useful value that can be used within discussion of physics - such as the energy deposited by a process or the value input for a calculation of total luminosity.

A. Differential Energy Flux

This term is used sometimes in plots, but it is a rarely used term.

$$\frac{dN}{dE} E = E \times M(E) \quad \left(\mathrm{cm^{-2}s^{-1}}\right) \tag{3}$$

Converting these units from CTools to VERITAS is easy: simply multiply this evaluated result by $1 \times 10^{-4} \times 1 \times 10^{-6}$ to convert meters and TeV.

B. Integral Energy Flux

This expression refers to the total amount of energy deposited within an energy range, which now is integrated. This is much more useful to physics because it can be converted to Luminosity for a given astrophysical source with known distance (i.e. Flux $= \frac{L}{4\pi d^2}$).

$$\int_{E_{min}}^{E_{max}} \frac{dN}{dE} E dE = \int_{E_{min}}^{E_{max}} M(E) E dE \quad \left(\mathrm{erg\,cm^{-2}s^{-1}}\right) \tag{4}$$

The units now have a new energy term *ergs*, which is by convention in the astro community. With this, we have to convert the value *after* the integral, simply with MeVtoErg $= 1.602 \times 10^{-6}$ or TeVtoErg $= 1.602$. Again, for VERITAS we can multiply this evaluated result by 1×10^{-4} to convert meters. However, we have to do the integral in TeV, which means conversion of the energy bounds and model/differential photon flux prefactor. Once meters and TeV is converted inside the integral, we can then apply TeVtoErg at the end after the integral.

Sometimes, this integral expression is referred to simply as *energy flux*. This is more common than using energy flux to mean differential energy flux. To be safe, explicitly say integral or differential every time.

III. SPECTRA

In most cases, spectral plots do not use any of these forms and instead convert energy fluxes to a form that is more clear for visually finding bumps and inspecting spectral shapes. Following the convention set by cosmic ray astronomy, (Fermi acceleration with respect to energy has the form of $\frac{dN}{dE} \propto E^{-p}$ for $p \geq 2$ and is the dominant process in many of these astrophysical systems) it is convention to plot

$$\frac{dN}{dE}|_{E_{mean}} \times E_{mean}^2 \quad \left(\mathrm{erg\,cm^{-2}s^{-1}}\right) \tag{5}$$

Note that this is a differential photon flux, evaluated at E_{mean}, times E_{mean}^2. In this case, the energy for which the $\frac{dN}{dE}$ is evaluated at is the geometric mean energy

$$E_{mean} = \sqrt{E_{min} \times E_{max}} \tag{6}$$

Note that the units for this plot is the same as the integral energy flux, which can lead to confusion.

For conversions: use the proper prefactor units (either in square centimeters or square meters, MeV or TeV) for your instrument and then multiply result by either MeVtoErg or TeVtoErg.

IV. UNCOMMON QUANTITIES

Although the following quantities are uncommon in high-energy astrophysics in general, they may occur in particular contexts.

A. Jansky

The jansky (Jy) is a non-SI unit of spectral flux density most commonly used in radio astronomy. It describes the emission from a source as a density of energy flux per frequency. This is useful for the radio band in which power is received over a detector bandwidth, as opposed to detecting individual photons. The unit is normalized to a small value useful for describing the weak fluxes from typical astronomical radio sources. In cgs units,

$$1 \text{ Jy} = 10^{-23} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1} \tag{7}$$

To convert a flux density S in janskys to energy flux, integrate S over the frequency range being measured. To convert the energy bounds $E_{\min}$, $E_{\max}$ in TeV to frequency bounds $\nu_{\min}$, $\nu_{\max}$ in Hz, multiply by $\approx 2.41799 \times 10^{26}$ Hz TeV^{-1} [This comes from $E = h\nu$, where h is Planck's constant in the proper units. Convert by multiplying or dividing.]. Then the integral energy flux is obtained by

$$10^{-23} \int_{\nu_{\min}}^{\nu_{\max}} S(\nu) d\nu \quad \left(\text{erg cm}^{-2} \text{ s}^{-1}\right) \tag{8}$$

In the case of VERITAS observations that are used in this Jansky context, use the energy range of the analysis. For example, some theoretical models predict flux density that needs to be converted to a flux observable with VERITAS, so convert with equation above but use the energy ranges applicable to that analysis.

B. Fluence

Fluence refers to the energy received per unit area, or equivalently, the energy flux integrated over time. This is a useful unit when studying transient events, in which the emission lasts for a short period of time. To calculate the fluence $\mathcal{F}$, integrate the energy flux F over the observation time.

$$\mathcal{F} = \int_{t_{\text{start}}}^{t_{\text{end}}} F(t) dt \quad \left(\text{erg cm}^{-2}\right) \tag{9}$$

V. USEFUL INTEGRALS

$$\int_{x_1}^{x_2} k_0 \left(\frac{x}{x_0}\right)^{-\gamma} \mathrm{dx} = \frac{k_0 x_0}{\gamma - 1} \left[\left(\frac{x_1}{x_0}\right)^{1-\gamma} - \left(\frac{x_2}{x_0}\right)^{1-\gamma}\right] \tag{10}$$

$$= \frac{k_0}{\gamma - 1} \left[x_1 \left(\frac{x_1}{x_0}\right)^{-\gamma} - x_2 \left(\frac{x_2}{x_0}\right)^{-\gamma}\right] \tag{11}$$

$$\int_{x_1}^{x_2} k_0 x \left(\frac{x}{x_0}\right)^{-\gamma} \mathrm{dx} = \frac{k_0 x_0^2}{\gamma - 2} \left[\left(\frac{x_1}{x_0}\right)^{2-\gamma} - \left(\frac{x_2}{x_0}\right)^{2-\gamma}\right] \tag{12}$$

$$= \frac{k_0}{\gamma - 2} \left[x_1^2 \left(\frac{x_1}{x_0}\right)^{-\gamma} - x_2^2 \left(\frac{x_2}{x_0}\right)^{-\gamma}\right] \tag{13}$$